Accelerator Concepts

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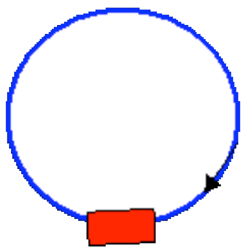
Introduction

- Goal is to produce luminosity for colliding beams program:

$$R = \frac{\Sigma_{int}}{A} \times N_1 \times (f \cdot N_2) = \Sigma_{int} \frac{f N_1 N_2}{A} = \Sigma_{int} \cdot L$$

  - Note: beam area *A* is product of accelerator properties and of beam properties

- Our model of accelerator:



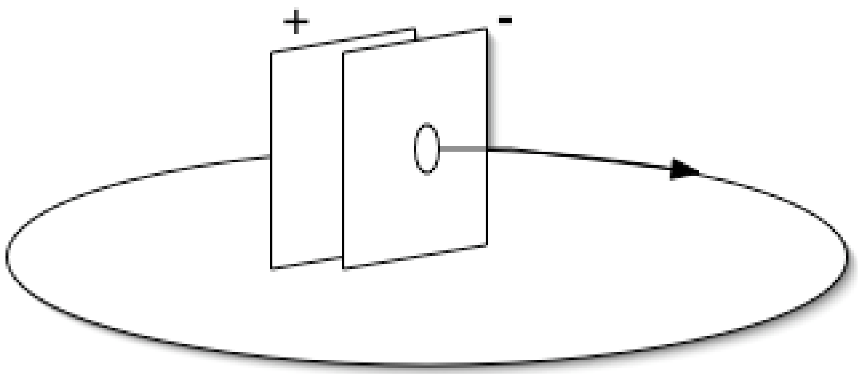
  - Accelerating device

    \+ magnetic field to bring it back to accelerate again

- 48,000 rev/sec x 3600 sec/hr x 20 hr/store = …
  - Need for ***Stability*** of particle motion!!!
- Will discuss (a) acceleration and longitudinal motion, and (b) transverse motion
  - *must show that is valid to treat them independently*

Acceleration and Longitudinal Motion

- DC acceleration can only take us so far
  - ~10 MeV, say, before electrostatic breakdown
- What if circulate particles back through the DC voltage again?
  - Net voltage along circuit will be zero; must turn voltage on/off --> AC



- Use resonant cavities, with electric field in z direction, $E_z = (V/g)\sin 2\pi f_{rf} t \qquad g = \textit{gap length}$
  - "*radio frequency*" cavities (10's - 100's MHz) typical

Acceleration (cont'd)

- Imagine particle with energy E_s revolving about accelerator with frequency f_0, and suppose the accelerating cavity field oscillates at $f_{rf} = f_0$. If the particle passes through the cavity at a phase ϕ_s, then the energy gain would be $\Delta E_s = eV \sin\phi_s$. Acceleration rate is then $$\frac{dE_s}{dt} = f_0\, eV\ \sin\phi_s$$
- If $f_{rf} = 2f_0$ then could imagine two particles (or groups of particles) located diametrically opposite each other which could be accelerated with the same system. In general, harmonic of the revolution frequency, $f_{rf} = hf_0$ allows one to have h possible accelerated groups. For Tevatron, *harmonic number* h = 1113, and f_{rf} = 53 MHz.
- Particle passing through cavity when cavity is at phase $\phi_s = \pi/2$ gives maximum energy gain per turn. However, particle arriving early gets less energy gain, and particle arriving late also gets less energy gain; no longitudinal "restoring force."
- Keeping ϕ_s in the "linear region" of the sine wave allows for longitudinal focusing, or "phase focusing" …

Phase Focusing

- In Tevatron, at 150 GeV, say, all particles moving at essentially $v = c$. Particle which arrives at the cavity at the same time as the ideal particle, but has too much energy *ΔE (or momentum, Δp)*, will follow a path with slightly larger radius (since the magnetic field does not bend it as much) and thus will end up lagging behind the ideal particle as it arrives at the cavity again. This particle needs to receive less energy than that of the ideal particle.
- Likewise, a particle with too little energy will take a slightly shorter path around the accelerator and thus will arrive at the cavity sooner; it needs to receive more energy than the ideal synchronous particle.
- Therefore, at injection, the optimal phase at which to inject is with the zero crossing of the RF sine wave at $\phi_s = \pi$ for the ideal particle; other particles nearby in energy will oscillate about this phase.
- Relative to the ideal synchronous particle, particles will oscillate in phase according to:

n = turn number

$$\frac{d^2\Delta\phi}{dn^2} = \left(\frac{2\pi h\eta}{\beta^2 E_s} eV\cos\phi_s\right)\sin\Delta\phi$$

Here, $\eta = 1/\gamma_t^2 - 1/\gamma^2$,

The Synchrotron

- Keeping $\phi_s = 0$, or $\phi_s = \pi$, implies no acceleration. Can vary acceleration rate by varying synchronous phase.
- Many earlier devices allowed for varying orbit radii as particles accelerated -- cyclotron, microtron, etc.
- *Synchrotron* -- use a time-varying magnetic field to maintain a constant orbital radius while the energy of the particle increases through the use of rf cavities
  - Begin with particle circulating with $\sin\phi_s = 0$ (no acceleration). If start to *adiabatically* increase the magnetic field, then phase-stable particles will continue to oscillate near their new synchronous phase as it changes.

Inj2accel.TB

Motion Near Synchronous Phase

- Ideal particle has revolution time $\tau = C/v$; then

$$\frac{d\tau}{\tau} = \frac{dC}{C} - \frac{dv}{v} \equiv (1/\gamma_t^2 - 1/\gamma^2)\,(dp/p) \equiv \eta(dp/p) = (\eta/\beta^2)(dE/E)$$

- The quantity γ_t, the "*transition gamma*," is a design property of the synchrotron
- So, we find that the phase and energy of a particle will evolve as:

(at entrance to cavity)

$$\begin{aligned} \Delta E_{n+1} &= \Delta E_n + eV(\sin\phi_n - \sin\phi_s) \\ \phi_{n+1} &= \phi_n + 2\pi h \cdot \frac{\eta}{\beta^2 E_s} \Delta E_{n+1} \end{aligned}$$

- Iterate and generate phase space plot…

Buckets.TB

Accel1.avi | Accel2.avi

Stability and Synchrotron Tune

- Combining the above difference equations, and taking the differential limit leads us back to the phase equation

$$\frac{d^2\Delta\phi}{dn^2} = \left(\frac{2\pi h\eta}{\beta^2 E_s} eV\cos\phi_s\right)\sin\Delta\phi \qquad \text{where } \Delta\phi = \phi - \phi_s.$$

- For small angles we have simple harmonic motion, with "frequency"

$$2\pi\nu_s = \sqrt{-\frac{2\pi h\eta}{\beta^2 E_s} eV\cos\phi_s}$$

- The quantity ν_s is the *synchrotron tune* and is the number of *synchrotron oscillations* which occur per revolution. At 150 GeV in the Tevatron, we get

$$\nu_s = \sqrt{-\frac{h\eta}{2\pi\beta^2 E_s} eV\cos\phi_s} = \sqrt{-\frac{1113\cdot 1/18^2}{2\pi(150\times 10^3)}(1)(-1)} \approx \frac{1}{500}$$

Phase Space

Buckets, Bunches, Batches, …

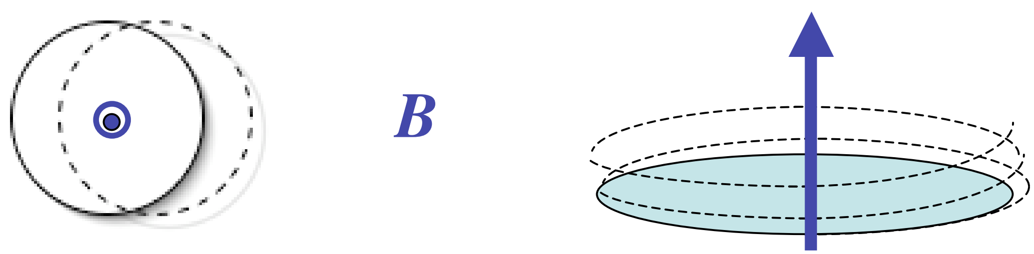
- Stable phase space region is called a *bucket*.
  - Boundary is the *separatrix*; only an approximation
  - $\phi_s = 0, \pi$ -- particles outside bucket remain in the accelerator

    "*DC beam*"
  - Other values of ϕ_s -- particles outside bucket are lost
    - DC beam from injection is lost upon acceleration
  - Motion near the unstable fixed points slows down; synchrotron tune depends upon oscillation amplitude (nonlinear; 'simple pendulum')
- *Bunches* of particles occupy buckets; but not all buckets need be occupied.
- *Batches* (or, bunch *trains*) are groupings of bunches formed in specific patterns, often from upstream accelerators

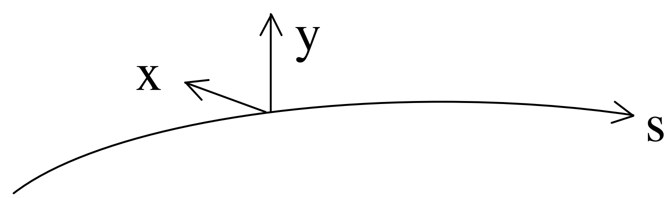
Transverse Motion

- So far, can see process for creating bunches of particles and accelerating to arbitrary energies; phase stability restores particles toward the ideal energy. However, not all particles (any??) travel along the ideal trajectory. Thus, need to also provide transverse restoring forces.
- Deflections in a Uniform magnetic field:

Horizontal -- stable　　　　Vertical -- NOT



- Electric Forces too weak at high energies; need to use magnetic fields to restore trajectories.
- Reference coordinate system:



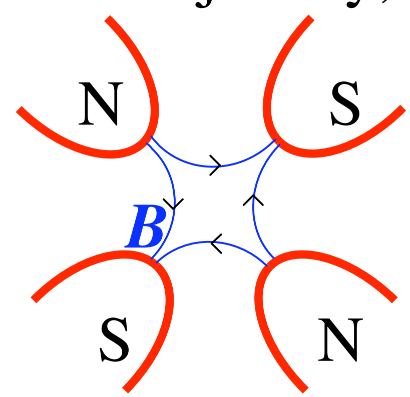
- Assuming small angles,

$$x' = dx/ds, \quad dx' = dp_x/p = F_x dt/p = F_x ds/(pv) = evB_y ds/(pv) = (eB_y/p)ds$$

Weak Focusing vs. Strong Focusing

- Original focusing schemes employed (now-called) Weak Focusing -- use outwardly wedged-shape pole faces on magnets to produce horizontal field which varies with vertical displacement
  - As accelerator radii increased, so did their required apertures -- and hence the required transverse real estate! (could *stand* inside some beam pipes!)
- Strong Focusing -- alternate the wedge direction of pole faces on successive magnets; makes field gradient vary from vertically focusing to vertically defocusing from magnet to magnet
  - Has a much stronger overall restoring force
- Eventually, realized could separate the focusing and bending functions -- uniform fields (dipoles) to bend the central trajectory, and quadrupole fields to provide a field gradient:

$$\vec{B} = B' \, (y \, \hat{x} + x \, \hat{y})$$

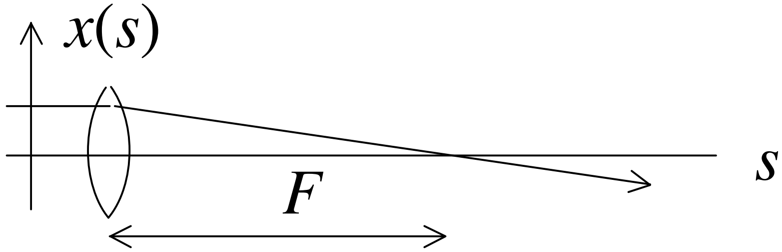


First *separated function* synchrotron: **Fermilab Main Ring**

Alternating Gradient Principle

- Think of standard focusing scheme as alternating system of focusing and defocusing lenses
- Quadrupole will *focus* in one transverse plane, but *defocus* in the other transverse plane; if alternate, can have net focusing in both planes
  - only if -- $F > L/2$ F = focal length, L = spacing
  - FODO cells: --- F ----- (-F) ----- F ----- (-F) ----- F ------ (-F) --
- Thin lens focal length:

$$\Delta x' = eB_y\ell/p = (eB'\ell/p)\,x \rightarrow 1/F = eB'\ell/p$$



Tevatron:

$$B' = 77\ \mathrm{T/m}, \quad \ell = 1.7\ \mathrm{m} \rightarrow F = 25\ \mathrm{m}$$

and $L = 30$ m

Stability Criterion

- Since can design system of lenses with appropriate separation and focal lengths, can decouple the radius of the synchrotron from the transverse aperture -- ***strong*** focusing. Thus, in principle, could build synchrotron with arbitrarily large circumference.
- Do not want all "lenses" equally spaced, necessarily. However, if system of linear restoring forces, then is linear algebra problem:
  - For N linear (restoring force) elements once around,

$$M_N M_{N-1} \cdots M_i \cdots M_2 M_1 \begin{pmatrix} x \\ x' \end{pmatrix}_n = M \begin{pmatrix} x \\ x' \end{pmatrix}_n = \begin{pmatrix} x \\ x' \end{pmatrix}_{n+1}$$

stable, if $|\mathrm{Trace}(M)| < 2$.

» Note: must analyze each plane separately, but for same lens system

Hill's Equation and the "Beta Function"

- Track thru elements (matrices); ex: FODO cells Envelope.TB
- Analytical Description:
  - Equation of Motion: $$\frac{dx'}{ds} = \frac{d^2x}{ds^2} = -\frac{eB'(s)}{p}x$$
    - or, $x'' + K(s)x = 0$ (Hill's Equation)
  - Nearly simple harmonic; assume solution: $x(s) = A\sqrt{\beta(s)}\sin[\psi(s)+\delta]$
  - Then, differentiating our solution twice, and plugging into Hill's Equation, we get …

$$\begin{aligned} x'' + K(s)x &= A\sqrt{\beta}\left[\psi'' + \frac{\beta'}{\beta}\psi'\right]\cos[\psi(s)+\delta] \\ &\quad + A\sqrt{\beta}\left[-\frac{1}{4}\frac{(\beta')^2}{\beta^2} + \frac{1}{2}\frac{\beta''}{\beta} - (\psi')^2 + K\right]\sin[\psi(s)+\delta] = 0 \end{aligned}$$

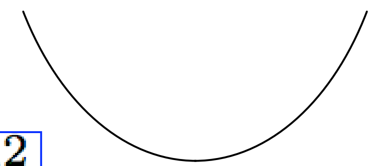
  - For arbitrary A, δ,
    - Must have $\beta > 0$, and first term --> $\psi''/\psi' = -\beta'/\beta \rightarrow \quad \psi' = 1/\beta$
    - Thus, second term implies differential equation for β:

Differentiating and simplifying: $\boxed{\beta''' + 4K\beta' + \beta K' = 0}$

Hill's Equation and Beta (cont'd)

- Typically, dK/ds = 0 in the design; so, $\beta''' + 4K\beta' = 0$
- In a "drift" region (no focusing),

$$\beta''' = 0$$



  - Thus, beta function is a parabola
    - If pass through a waist at $s = 0$, then, $\boxed{\beta(s) = \beta^* + \frac{s^2}{\beta^*}}$

- Through focusing region (quad. magnet, say), $K = const$

$$\beta'' + 4K\beta = const.$$

  - Thus, beta function is a sin/cos or sinh/cosh function, with an offset
    - "driven harmonic oscillator," with constant driving term

- So, optical properties of synchrotron (β) are now decoupled from particle properties (A, δ) and accelerator can be designed in terms of optical functions; beam size will be proportional to $\beta^{1/2}$

Tune

- Since $x(s) = A\sqrt{\beta(s)}\sin[\psi(s)+\delta]$ and $\psi' = 1/\beta$

  then the total phase advance around the circumference is given by

  $$\psi_{tot} \equiv 2\pi\nu = \oint \frac{ds}{\beta}$$

  The tune, ν, is the number of "*betatron oscillations*" per revolution

  The phase advance through one FODO cell in the Tevatron is given by

  $$\psi_{cell} = 2\sin^{-1}\left(\frac{L}{2F}\right)$$

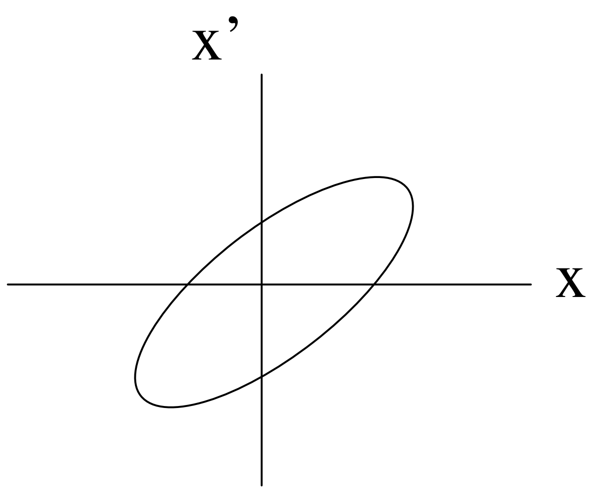
  For Tevatron, $L/2F = 0.6$, and since there are about 100 cells, the total tune is about 100 x (2 x 0.6)/2π ~ 20

- Note: since betatron tune ~ 20, and synchrotron tune ~ 0.002, it *is* (relatively) safe to consider these effects independently

Emittance

- Just as in longitudinal case, we look at the phase space trajectories, here *x*-*x*', in transverse space.
- Viewed at one location, phase space trajectory of a particle is an ellipse:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$



Here,

$$\alpha \equiv -\frac{1}{2}\beta'$$

$$\gamma \equiv \frac{1+\alpha^2}{\beta}$$

α, β, γ are the *Courant-Snyder* parameters

While beta function changes along the circumference, the area of the phase space ellipse = πA^2, and is independent of location!
So, define *emittance,* ε, of the beam as area of phase space ellipse containing some fraction (95%, say) of the particles (units = mm-mrad)

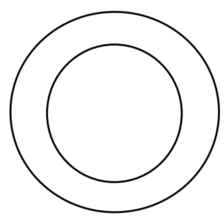
Emittance (cont'd)

- Emittance of the particle distribution is thus a measure of beam quality.
  - At any one location $$\langle x^2\rangle(s) = \sqrt{\epsilon\beta(s)}$$
  - note: β in m, ε in mm-mrad; then x in mm
- Variables x,x' are not canonical variables; but x,p_x are; the area in x-p_x phase space is an adiabatic invariant; so, define a *normalized emittance* as
$$\epsilon_N = \epsilon \cdot (\gamma v/c)$$
- The normalized emittance should not change as we make adiabatic changes to the system (accelerate, for instance). This implies that the beam size will shrink as $p^{-1/2}$.
- We define a "95% emittance" as the area which contains 95% of a Gaussian beam, for which $\sigma = x_{rms}$
  - Then the 95% normalized emittance will be: $$\epsilon_N = 6\pi \frac{\langle x^2\rangle(s)}{\beta(s)}(\gamma v/c)$$

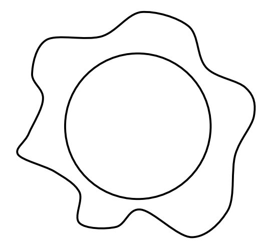
Off-Momentum Considerations

- Particle orbit radius in a uniform magnetic field depends upon the particle momentum. Thus, since higher momentum means larger radius, an ensemble of particles with various momenta distributed about the ideal momentum will be dispersed horizontally onto different orbits.

Uniform:



Synchrotron:



- But, the higher momentum orbits will be distorted due to the focusing fields encountered. The exact shapes of these orbits will depend upon the arrangement of focusing magnets. These orbits are described by the *Dispersion Function*:

$$D(s) = \Delta x_{c.o.}(s)/(\Delta p/p)$$

Momentum Effects (cont'd)

- Thus, total beam size will depend upon the Dispersion function as well:

$$\langle x^2 \rangle = \frac{\epsilon_N \beta(s)}{6\pi\gamma} + D(s)^2 \langle (\Delta p/p)^2 \rangle$$

- So, make Dispersion small (= 0!) at IP, for instance
- From Hill's Equation, we see that the "spring constant" depends upon momentum, and thus the tune (betatron frequency) will also…

  – *Chromaticity* = $\xi \equiv \Delta\nu/(\Delta p/p)$

- Lastly, path length change due to momentum:

  (loose end, from earlier…)

$$dC/C \equiv 1/\gamma_t^2 = \langle D/\rho \rangle$$

Chromaticity Adjustment

- To give all particles the same tune, regardless of momentum, need capability of generating a "gradient" which depends upon momentum. Since the beam orbits spread out horizontally due to momentum, can use a sextupole field

$$\vec{B} = \frac{1}{2} B''[2xy\ \hat{x} + (x^2 - y^2)\ \hat{y}]$$

which gives

$$\partial B_y / \partial x = B'' x = B'' D(\Delta p / p)$$

i.e., a gradient which depends upon momentum

- Use sextupole magnets to control the chromaticity; but, now introduces a nonlinear transverse field, and all that implies!

Sextupole.TB

Other Effects to Consider …

- Linear errors
  - Steering errors and corrections; integer resonance
  - focusing errors and corrections; half-integer resonance
- Further field imperfections (nonlinearities)
  - Dynamic aperture, resonance conditions, …
  - Resonant extraction
- Coupling of degrees-of-freedom
  - Transverse linear coupling
  - Nonlinear coupling
  - Synchro-betatron coupling
- Emittance growth mechanisms
- Synchrotron Radiation
- Space charge interactions (mostly low-energies)
- Beam-beam interactions
  - Head-on collisions, long-range interactions
- Impedance / Wake Fields due to pipe, “cavities,” …
  - Coherent instabilities -- Head-tail, resistive wall, etc.
- RF Manipulations --
  - coalescing, barrier buckets, cogging, slip stacking, …
- Emittance reduction -- stochastic cooling, electron cooling
- …

Help is always *welcome* !!

Back to Luminosity

- Round, uniform bunches: $$L = \frac{fN_1N_2}{A} = \frac{f_0BN_1N_2}{A}$$
  - *but, aren't really uniform…*
- Round Gaussian bunches: $$L = \frac{f_0BN_1N_2}{4\pi\sigma^2} = \frac{3f_0BN_pN_{\bar{p}}\gamma}{2\beta^*\epsilon_N}$$
  - *but, $\beta = \beta^* = const.$ not realistic…*

    $1+(40/35)^2 = 2.3$
- Hourglass: $$L = \frac{3f_0(BN_{\bar{p}})N_p\gamma}{2\beta^*\epsilon_N} \cdot H(\beta^*, \sigma_z)$$
- Tevatron Parameters:

$$L = \frac{3(48\text{kHz})(180 \cdot 10^{10})(3 \cdot 10^{11})(1000)}{2(35\text{cm})(25\pi \cdot 10^{-4}\text{cm})} \cdot (0.6) = 80 \times 10^{30}\ \text{cm}^{-2}\text{sec}^{-1}$$

  - Note: ignoring diff's in ε, σ_z, etc. for p's ,pbar's, crossing angle, etc.

References

- D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators*, John Wiley & Sons (1993)
- S. Y. Lee, *Accelerator Physics*, World Scientific (1999)
- E. J. N. Wilson, *An Introduction to Particle Accelerators*, Oxford University Press (2001)

  and many others…

- Particle Accelerator Schools --
  - USPAS: http://uspas.fnal.gov
  - CERN CAS: http://cas.web.cern.ch

- Conference Proceedings --
  - Particle Accelerator Conference (2003, 2001, …)
  - European Particle Accelerator Conference (2004, 2002, …)